

Microscopic construction of the chiral Luttinger liquid theory of the quantum Hall edge.

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We give a microscopic derivation of the chiral Luttinger liquid theory (χ LL) for the Laughlin states. Starting from the wave function describing an arbitrary incompressibly deformed Laughlin state (IDLS) we quantize these deformations. In this way we obtain the low-energy projections of local microscopic operators and derive the quantum field theory of edge excitations directly from quantum mechanics of electrons. This shows that to describe experimental and numeric deviations from χ LL one needs to go beyond Laughlin's approximation. We show that in the large N limit the IDLS is described by the dispersionless Toda hierarchy.

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The fractional quantum Hall effect is a manifestation of strong correlations in the two dimensional electron gas in a strong magnetic field [1]. At certain filling fractions $\nu = 2\pi l^2 \rho$, where l is the magnetic length and ρ is the electron density, the electrons “condense” into an incompressible fluid, which leads to the quantization of the Hall conductance. While the spectrum of bulk excitations of the incompressible Hall state is gapful, the boundary of such a state exhibits rich low-energy physics associated with gapless edge modes. In 1990, based on elegant arguments of locality, chirality and gauge invariance, X.-G. Wen proposed an effective field theory for the $\nu = 1/(2p+1)$ quantum Hall edge [2]. Later on, this theory was generalized to other filling factors at which the incompressible quantum Hall states occur [3, 4]. The construction given in [2], is usually referred to as the chiral Luttinger liquid (χ LL). The χ LL theory predicts that once the Hall conductance of the incompressible bulk state is known, the low-energy properties of the edge become independent of the details of electron-electron interactions and the confining potential, the only non-universal parameter being the propagation velocity of the edge excitations. Probably, the most intriguing prediction of the χ LL theory is the universal power law scaling of local operators and in particular the electron spectral weight at the edge.

Early attempts to put the χ LL theory to the test were quite successful. The compressible edge modes were observed in a number of experiments. The low energy excitation spectrum studied by exact diagonalization of small systems at $\nu = 1$ [5] and at $\nu = 1/3$ [6] was found to be consistent with the predictions of χ LL. The ground state momentum distribution function was tested numerically for the Laughlin state in various geometries [7, 8] and was found to be in agreement with the predictions of χ LL. Recently, however, both experimental [9] and numerical [10, 11, 12] evidences emerged that the scaling properties

of the observables at the quantum Hall edge may actually be non-universal. Moreover, in a recent work [13] very interesting numerical results were reported, indicating that one of the key assumptions of the χ LL theory, namely the assumption that the electron creation (annihilation) operators preserve their fermionic statistics in the low-energy effective theory, may be violated for some quantum Hall states. This results question the universality of the χ LL theory and motivate a search for microscopic approaches to the description of the quantum Hall edge. This task can be pursued in two different ways. One is to use the composite fermion picture [14]. The problem of this approach is that at some point it has to rely on the mean-field approximation, which, if valid, is still extremely hard to justify. Another way is to use trial wave functions to build the low energy sector of the Hilbert space and to construct the low energy projections of local operators explicitly. Such a construction was discussed in [15, 16], where the low-energy projections of the edge current operators were studied. In [16] a local operator for the periphery deformation accommodating a unit charge was also proposed. In this paper we use a similar philosophy to give a direct (rather than phenomenological) derivation of the χ LL theory, starting from quantum mechanical many-particle wave functions. Here it will be done for the Laughlin state. From our construction it becomes clear that one needs to go beyond Laughlin's approximation to understand the experimental and numerical deviations from the χ LL theory. At the same time, being microscopic in nature, our approach introduces a framework for field-theoretical treatment of corrections to the Laughlin's approximate ansatz.

One way to view the χ LL theory of the quantum Hall edge is to consider it as a dynamical description of the low-energy projections of the microscopic operators. Formally, for a given operator A its low energy projection,

which we will denote as \hat{A} , is defined as

$$\hat{A} = \mathcal{P}A\mathcal{P}, \quad \mathcal{P} = \sum_{|\nu\rangle \in \mathcal{H}} |\nu\rangle\langle\nu|, \quad (1)$$

where \mathcal{H} is the Hilbert space of such low-energy excitations. The χ LL construction [2, 3] relies on some assumptions about the low energy projection of microscopic operators. Those include: chirality; current algebra; charge and statistic of the projected fermion operators. Neither reliability nor limitations of any of these assumptions can be understood without the understanding of the properties of the low energy projection. In the following paragraphs we use trial wave functions for a consistent microscopic construction of the projected currents and the projected local electron operator and check their physical properties for the Laughlin states $\nu = 1/(2p+1)$.

We base our considerations on the N -particle Laughlin's wave function [17]

$$\langle z_1, \dots, z_N | 0 \rangle = \frac{1}{\sqrt{N!}} \prod_{i < j} (z_j - z_i)^{\frac{1}{\nu}} \prod_{j=1}^N e^{-\frac{1}{2h\nu} z_j \bar{z}_j} \quad (2)$$

which is known to describe the ground state $|0\rangle$ of a disk-shaped droplet of the quantum Hall fluid with remarkable accuracy [18]. The parameter $h = 2l^2/\nu$ was introduced in Eq. (2) to shorten notation. Laughlin pointed out [1] that by the following deformation of the ground state:

$$\langle z_1, \dots, z_N | t_k \rangle = \langle z_1, \dots, z_N | 0 \rangle \prod_{j=1}^N e^{\frac{1}{h\nu} \omega(z_j)}, \quad (3)$$

the shape of the quantum Hall droplet can be changed in such a way as to preserve the electron density and the area of the droplet. In Eq. (3)

$$\omega(z) \equiv \sum_{k>0} t_k z^k \quad (4)$$

is an entire function of z depending on the complex parameters t_k . Below, we will often refer to the state Eq. (3) as to the *incompressibly deformed Laughlin state* (IDLS). The geometry of the IDLS for $\nu = 1$ was investigated in the recent work [19]. It was explicitly shown that in the large N limit the charge density, defined as $\rho(\xi, \bar{\xi}) = \text{const} \int d\mu |\langle z_1, \dots, z_{N-1}, \xi | t_k \rangle|^2$, where $d\mu \equiv d^2 z_1 \dots d^2 z_N$, is given by the two-dimensional step-function, with the support in the region, bounded by the curve γ whose harmonic moments are equal to t_k from Eq. (4), i.e.

$$\oint_{\gamma} \frac{dz}{2\pi k} \frac{\bar{z}}{z^k} = -\frac{1}{\pi k} \int_{D_-} z^{-k} d^2 z = t_k \quad (5)$$

(here D_- stands for the *exterior* of the droplet with the boundary γ , $D_+ = \mathbb{C}/D_-$ is an *interior* of the domain).

In fact, it is also true for $\nu < 1$ [20]. The key result is that the norm of the IDLS

$$\tau_N^\nu(t_k, \bar{t}_k) = \int d\mu |\langle z_1, \dots, z_N | t_k \rangle|^2 \quad (6)$$

is given in the large N limit ($N \rightarrow \infty$, $h \rightarrow 0$, $t_0 = Nh$ is being kept fixed) by

$$\tau_N^\nu(t_k, \bar{t}_k) \rightarrow \tau^\nu(t_0, t_k, \bar{t}_k) = e^{h^{-2}\nu^{-1}F(t_0, t_k, \bar{t}_k)}, \quad (7)$$

where F is the logarithm of the dispersionless tau-function of 2D Toda lattice hierarchy [21]. Note, that the function F is *the same* for both $\nu = 1$ and $\nu < 1$. It will be important that for $t_k, \bar{t}_k = 0$ (from now on we choose $t_0 = 1$ unless explicitly stated otherwise)

$$\frac{\partial F}{\partial t_p} = \frac{\partial F}{\partial \bar{t}_p} = 0 \quad \text{and} \quad \frac{\partial^2 F}{\partial t_q \partial \bar{t}_p} = q \delta_{q,p} \quad (8)$$

Next, we calculate the energy of the IDLS. Note that the incompressible deformation does not influence the short-range exchange-correlation part of the interaction energy since the local structure of the wave function is unaffected. Thus the only change of the energy comes from the Hartree term: $E(t_k, \bar{t}_k) = \iint_{D_+} d^2 z d^2 w V(|z - w|) + \int_{D_+} d^2 z U(|z|)$. Here $V(r)$ is the electron-electron repulsion potential and $U(r)$ is the axially symmetric confining potential, which keeps the Laughlin state from breaking apart. The Hilbert space \mathcal{H}_N of the low-energy excitations of the N -particle incompressible Laughlin droplet is defined as that of IDLS with the energy less than some cutoff Λ . For such states one has

$$E(t_k, \bar{t}_k) = E_0 + \sum_{k>0} s(k) k^2 t_k \bar{t}_k, \quad \sum_{k>0} s(k) t_k \bar{t}_k < \Lambda \quad (9)$$

where $s(k)$ is given by the following expression:

$$s(k) = \frac{\pi}{h^2} U_0 k^2 + \int_0^{2\pi} d\theta V(2|\sin \theta|) [\cos k\theta - \cos \theta] \quad (10)$$

For small k Eq. (10) gives $s(k) = s_0 k^2$ where s_0 is a *Fermi velocity*.

The projection of an arbitrary state $\Phi(z_1, \dots, z_N)$ in \mathcal{H}_N onto the incompressibly deformed Laughlin state $\Psi(z_1, \dots, z_N | t_k)$ defined in Eq. (3)

$$\Phi(\bar{t}_k) = \int d\mu \bar{\Psi}(\bar{z}_1, \dots, \bar{z}_N | \bar{t}_k) \Phi(z_1, \dots, z_N) \quad (11)$$

defines the \bar{t} -representation of the state Φ . As we will see, it is particularly convenient for building the low-energy projections of the microscopic operators. It is important to know how the inner product of two states in the \bar{t} representation is calculated. Introduce the following basis of states

$$|\vec{n}\rangle = \frac{1}{\sqrt{\tau^\nu}} \prod_{k=1}^{\infty} \left(\frac{\partial}{\partial t_k} \right)^{n_k} \Psi_N(z_1 \dots z_N | t_k) \Big|_{t_k=0}, \quad (12)$$

where $\vec{n} = (n_1, \dots, n_p, \dots)$ is a vector with non-negative integer valued components. Here τ^ν in the denominator is the shorthand for $\tau^\nu(t_0, 0, 0)$. Since any entire function of t_k can be expanded in Taylor series, the states Eq. (12) form an (over)complete basis in the subspace \mathcal{H}_N . The Gram matrix of the basis Eq. (12) is calculated as

$$\langle \vec{n} | \vec{m} \rangle = \frac{1}{\tau^\nu} \prod_{k,p \geq 1} \frac{\partial^{m_k+n_p} \tau^\nu(t_k, \bar{t}_k)}{\partial t_k^{m_k} \partial \bar{t}_k^{n_p}} \Big|_{t_k, \bar{t}_k=0} \quad (13)$$

By using the asymptotic formulas Eqs. (7), (8) one finds that to the leading order in \hbar basis (12) is orthogonal [22]

$$\langle \vec{n} | \vec{m} \rangle = C_{\vec{n}} \delta_{\vec{m}, \vec{n}}, \quad \text{where} \quad C_{\vec{n}} = \prod_{k \geq 1} \left(\frac{k}{\nu \hbar^2} \right)^{n_k} n_k! \quad (14)$$

From definitions Eqs. (11) and (12) it follows that the projection of an arbitrary state Φ in \bar{t} -representation onto the state $|\vec{n}\rangle$ is given by

$$\langle \vec{n} | \Phi \rangle = \frac{1}{\sqrt{\tau^\nu}} \prod_{k \geq 1} \left(\frac{\partial}{\partial \bar{t}_k} \right)^{n_k} \Phi(\bar{t}_k) \quad (15)$$

Therefore, for two states Φ_a and Φ_b we have

$$\langle \Phi_a | \Phi_b \rangle = \frac{1}{\tau^\nu} \sum_{\vec{n}} \prod_{k \geq 1} \left(\frac{\partial^2}{\partial t_k \partial \bar{t}_k} \right)^{n_k} \frac{\bar{\Phi}_a(t_p) \Phi_b(\bar{t}_p)}{\langle \vec{n} | \vec{n} \rangle} \quad (16)$$

Using Eq. (14) one can see that the following compact representation of Eq. (16) exists

$$\langle \Phi_a | \Phi_b \rangle = \bar{\Phi}_a \left(\frac{\hbar^2 \nu}{k} \frac{\partial}{\partial t_k} \right) \Phi_b(\bar{t}_k) \Big|_{\bar{t}_k=0} \quad (17)$$

The algebra of operators in \bar{t} -representation is generated by the elements

$$\hat{a}_{-k} = \sqrt{\frac{k}{\nu \hbar^2}} \bar{t}_k, \quad \hat{a}_k = \sqrt{\frac{\nu \hbar^2}{k}} \frac{\partial}{\partial \bar{t}_k}, \quad [\hat{a}_k, \hat{a}_{-k}] = 1 \quad (18)$$

(for $k > 0$). From the inner product (17) one finds

$$\hat{a}_{-k} = \hat{a}_k^\dagger \quad (19)$$

The \bar{t} representation has a natural extension to the low energy Fock space $\mathcal{H} = \mathcal{H}_N \oplus \mathcal{H}_{N-1} \oplus \mathcal{H}_{N+1} \oplus \dots$. In this space $t_0 = \hbar N$ becomes an operator rather than a parameter. In the large N limit one can neglect the constraint $t_0 > 0$ and realize the particle number operator \hat{t}_0 as the angular momentum operator conjugate to an auxiliary angle variable φ satisfying $\varphi = \varphi + 2\pi$:

$$\hat{t}_0 = -i\hbar \frac{\partial}{\partial \varphi}, \quad [\hat{t}_0, e^{\pm i\varphi}] = \pm \hbar e^{\pm i\varphi} \quad (20)$$

Consider some N -particle IDLS $|t_0, t_k\rangle$, where $t_0 = \hbar N$ explicitly shows the number of particles. This is an eigenstate of \hat{t}_0 . Any state $|\Phi\rangle$ where the number of particles

is not defined can be projected on $|t_0, t_k\rangle$. This projection defines the \bar{t} representation in \mathcal{H} : $\Phi(t_0, \bar{t}_k) = \langle t_0, t_k | \Phi \rangle$. For example, the \bar{t} representation of a N -particle IDLS characterized by the harmonic moments t'_k will be given by

$$\langle t_0, t_k | t'_0, t'_k \rangle = \tau^\nu(t_0, t'_k, \bar{t}_k) \delta_{t_0, t'_0} \quad (21)$$

In the following we will use the explicit action of the phase operators $e^{i\varphi}$ in the \bar{t} representation:

$$e^{i\varphi} \Phi(t_0, \bar{t}_k) = \Phi(t_0 - \hbar, \bar{t}_k). \quad (22)$$

This formula completes the description of the \bar{t} -representation of the space \mathcal{H} of low-energy excitations.

It is easy to see that for positive m \hat{a}_m is proportional to the Fourier component $\hat{\rho}_m$ of the edge density operator [2, 16] defined as $\rho_m = (z_1^m + \dots + z_N^m)/\sqrt{2\pi}$. Namely from Eqs. (18), (3) and (4) one finds

$$\hat{\rho}_m = \hat{a}_m \sqrt{\frac{\nu|m|}{2\pi}} \quad \text{and} \quad [\hat{\rho}_m, \hat{\rho}_n] = \frac{\nu m}{2\pi} \delta_{m+n,0} \quad (23)$$

(By virtue of Eq. (19), Eq. (23) is true for both positive and negative m). Thus we have derived microscopically the central charge of the current algebra of the edge density operators, obtained in [2] from the gauge invariance arguments.

Consider now the microscopic fermion field $\psi(\zeta)$, which annihilates an electron at the point ζ . In z representation it is defined as

$$[\psi(\zeta)\Psi](z_1, \dots, z_N) = \sqrt{N+1} \Psi(z_1, \dots, z_N, \zeta) \quad (24)$$

We will find the action of the conjugate (creation) operator ψ^\dagger on the \bar{t} representation Eq. (11) of the state $|t'_0, t'_k\rangle$, given by Eq. (21). The \bar{t} representation of the state $\psi^\dagger(\bar{z})|t'_0, t'_k\rangle$ is given by

$$\hat{\psi}^\dagger(\bar{\zeta}) \tau^\nu(t_0, t'_k, \bar{t}_k) \delta_{t_0, t'_0} = \langle t_0, t_k | \psi^\dagger(\bar{\zeta}) | t'_0, t'_k \rangle. \quad (25)$$

where $\hat{\psi}^\dagger$ on the l.h.s. is by definition the \bar{t} representation of the low energy projection of the fermion operator. We start from calculating the matrix element

$$\begin{aligned} \langle t_0, t_k | \psi^\dagger(\bar{\zeta}) | t'_0, t'_k \rangle &= \\ e^{-\frac{|\zeta|^2 - 2\omega(\bar{\zeta})}{2\hbar\nu}} \delta_{t_0 - \hbar, t'_0} \int d\mu \prod_{j=1}^N (\bar{\zeta} - \bar{z}_j)^{\frac{1}{\nu}} |\langle z_1 \dots z_N | t_k \rangle|^2 \\ &= \bar{\zeta}^{\frac{t_0 - \hbar}{\hbar\nu}} e^{-\frac{|\zeta|^2 - 2\omega(\bar{\zeta})}{2\hbar\nu}} \delta_{t_0 - \hbar, t'_0} \times \tau^\nu \left(t_0 - \hbar, t_k, \bar{t}_k - \frac{\hbar}{k\zeta^k} \right) \end{aligned} \quad (26)$$

where $t_0 = \hbar(N+1)$. The last line in Eq. (27) was obtained using the definitions Eqs. (6) and (7), exponentiating $(\bar{\zeta} - \bar{z}_j)^{\frac{1}{\nu}}$ and expanding the logarithm. Because of the fact that $\tau^\nu(t_0, t_k, \bar{t}_k)$ is a function, analytic in both sets t_k and \bar{t}_k independently, Eq. (27) actually gives the

r.h.s. of Eq. (25) for arbitrary t'_k . This defines the action of the operator $\hat{\psi}^\dagger$ in the \bar{t} representation. Indeed, comparing Eqs. (25), (27) and (22) we get

$$\hat{\psi}^\dagger(\bar{\zeta}) = e^{-\frac{1}{2h\nu}|\zeta|^2} e^{i\varphi} : e^{\hat{\phi}(\bar{\zeta})} : \quad (27)$$

where

$$\hat{\phi}(\bar{\zeta}) = \frac{\hat{t}_0}{h\nu} \ln(\bar{\zeta}) + \sum_{k \geq 1} \left[\frac{1}{\sqrt{\nu k}} \bar{\zeta}^k \hat{a}_k^\dagger - \frac{1}{\sqrt{\nu k}} \bar{\zeta}^{-k} \hat{a}_k \right] \quad (28)$$

The normal ordering $::$ is understood in the standard sense $: \exp(\hat{\phi}) := \exp(\hat{\phi}_0) \exp(\hat{\phi}_+) \exp(\hat{\phi}_-)$, where $\hat{\phi}_+$ is the positive frequency part of $\hat{\phi}$, only containing creation operators \hat{a}_k^\dagger , $\hat{\phi}_-$ is the negative frequency part and $\hat{\phi}_0$ is the zero mode. The field $\hat{\phi}$ satisfies the obvious property $[\hat{\phi}(\bar{\zeta})]^\dagger = -\hat{\phi}(1/\bar{\zeta})$. Therefore the electron annihilation operator is

$$\hat{\psi}(\zeta) = e^{-\frac{1}{2h\nu}|\zeta|^2} : e^{-\hat{\phi}(1/\zeta)} : e^{-i\varphi}. \quad (29)$$

This completes the construction of the projected fermion operators.

Notice that since $|t'_0, t_k\rangle$ span the whole Hilbert space \mathcal{H} , we have defined the action of the operator $\hat{\psi}^\dagger(\bar{\zeta})$ on any state in \mathcal{H} . In this way one can in principle find \bar{t} representation of any microscopic operator.

Derive, for instance, the \bar{t} representation of the Hamiltonian. By virtue of Eq. (9) and with the help of the relations Eq. (7), (8) one has

$$\langle t_k | (H - E_0) | t_k \rangle = s_0 h^2 \nu \sum_{k \geq 1} k \bar{t}_k \frac{\partial}{\partial \bar{t}_k} \tau^\nu(t_k, \bar{t}_k) \quad (30)$$

According to the given above prescription the projected Hamiltonian is immediately identified as

$$\hat{H} - E_0 = s_0 h^2 \nu \sum_{k \geq 1} k \hat{a}_{-k} \hat{a}_k \quad (31)$$

Note, that for the \bar{t} -representation of the vacuum (Laughlin) state, $\tau^\nu(t_0, 0, \bar{t}_k) = \text{const}$ we have

$$\hat{a}_k |0\rangle = 0, \quad k > 0, \quad (32)$$

therefore the operator Eq. (31) is normal ordered. Notice that Hamiltonian Eq. (31) coincides with the Virasoro generator L_0 of the theory of free chiral scalar ϕ . From Eq. (31) one can see that states $|\vec{n}\rangle$ are indeed the eigenstates of the projected Hamiltonian

$$(\hat{H} - E_0) |\vec{n}\rangle = s_0 h^2 \nu \sum_{k \geq 0} k n_k |\vec{n}\rangle \quad (33)$$

The positive- and the negative-frequency parts of $\hat{\phi}$ satisfy the following commutation relations

$$[\hat{\phi}_-(z), \hat{\phi}_+(w)] = - \sum_k \frac{1}{\nu k} \left(\frac{w}{z} \right)^k = \frac{1}{\nu} \ln \left(1 - \frac{w}{z} \right) \quad (34)$$

The correlation functions of the projected fermion operators can be obtained in a standard way from Eqs. (27), (29), (32), (34) and with the help of the formula

$$: e^{\hat{\phi}(\bar{z})} :: e^{-\hat{\phi}(w)} := : e^{\hat{\phi}(\bar{z}) - \hat{\phi}(w)} : e^{-[\hat{\phi}_-(\bar{z}), \hat{\phi}_+(w)]}. \quad (35)$$

For large enough distances ($|z - w| > h^2 s_0 / \Lambda$) one has

$$\langle 0 | \psi^\dagger(\bar{z}) \psi(w) | 0 \rangle = e^{-\frac{1}{2h\nu}(|w|^2 + |z|^2)} \frac{(w\bar{z})^{\frac{N}{\nu}}}{(1 - \frac{1}{w\bar{z}})^{1/\nu}}. \quad (36)$$

For $|z| = |w| = 1$ this formula is very similar to Wen's result [2]. The difference between the correlation function proposed by Wen and Eq. (36) is in the boundary conditions. While Wen's correlation function corresponds to antiperiodic boundary conditions for the fermion field $\psi(\theta)$, the microscopically derived correlation function is periodic in θ , which is more natural.

To calculate the time-dependent correlation function, we use the Hamiltonian Eq. (31). One can easily see that

$$e^{i\hat{H}t} \hat{\phi}(z) e^{-i\hat{H}t} = \hat{\phi}(ze^{i\omega t}) \quad (37)$$

where $\omega = h^2 \nu s_0$ (this formula is the microscopically derived chirality condition for the field $\hat{\phi}$). Thus

$$\langle 0 | \psi^\dagger(\bar{z}, t) \psi(w) | 0 \rangle = \langle 0 | \psi^\dagger(\bar{z} e^{-i\omega t}) \psi(w) | 0 \rangle. \quad (38)$$

This result also agrees with Wen's theory. Finally, using the Campbell-Hausdorff formula and Eq. (22) for the product of two fields Eq. (29) one finds

$$\hat{\psi}(z) \hat{\psi}(w) = \left(\frac{w}{z} \right)^{\frac{1}{\nu}} e^{[\hat{\phi}(1/z), \hat{\phi}(1/w)]} \hat{\psi}(w) \hat{\psi}(z). \quad (39)$$

This formula is correct for large enough separations $|z - w|$. Calculating the commutator in the exponential with the help of Eq. (34) one finds

$$\hat{\psi}(z) \hat{\psi}(w) + \hat{\psi}(w) \hat{\psi}(z) = 0. \quad (40)$$

In conclusion, we performed an explicit construction of the projections of the microscopic operators onto the space of incompressible deformations of the Laughlin state. As a main technical tool we used the exact large N asymptotic for the norm of the IDLS. It is given by the dispersionless limit of the integrable Toda hierarchy. We explicitly built both negative and positive modes of the edge density operator and obtained the central charge of their current algebra. We found that the fermion operators projected onto the low-energy subspace of the space of IDLS have the properties predicted by the χ LL theory. Thus, our main result is in establishing the one-to-one correspondence between the χ LL and the space of incompressible deformations of the Laughlin state. Understanding the experimental and numerical deviations from the χ LL should be sought in non-trivial corrections

to the Laughlin state, which in our formalism amounts to expanding the Hilbert space of the edge states and constructing the non-linear field theory in this bigger space. This work is planned for the future.

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